Charging and Discharging Process of a Capacitor

Objectives:

- Observe and understand the charging and discharging process of a capacitor;
- Build electric circuits with various resistors and capacitors connected to the Raspberry Pi in order to visualise, record and analyse the charging and discharging process of capacitors;
- Measure the voltage across the capacitor and the charging and discharging current in the RC-circuit using the Analog-to-Digital Converter ADS1115;
- Use the PhyPiDAQ-Software to display specific quantities such as the voltage across the capacitor or the stored charge in real-time and in various graphical form;
- Employ spreadsheets like LibreOffice or Excel to process the measurements stored in .csv files to derive the time constant of a RC circuit;
- Gain an understanding of the difference between the voltage across the capacitor, electric charge on the capacitor and charging/discharging current.
- Interpret and compare the voltage on capacitor versus time graphs and the time constant related to these graphs for different types of circuits such as:
    - RC circuits with increasing values of capacitance and resistance;
    - RC circuits with capacitors connected in series or in parallel;
    - RC circuits with resistors connected in series or in parallel;
- Use the measurements stored in .csv in interdisciplinary ways, like for instance in Maths for modelling certain growth and decay processes based on exponential functions;
- Gain a deeper insight into mutual relationships between the voltage on capacitor displayed in real-time on the measurement window and the exponential function of voltage against time, which is the solution of the differential equation deduced from the Kirchhoff's loop law.
- Use the measurements stored in .csv to show that the stored energy on capacitor doesn't depend on the resistance of the RC circuit.

Materials and Equipment:

Raspberry Pi 3B+ or 4B	Resistor of 1,0kΩ, 10kΩ , 100kΩ
PhyPiDAQ-Software for Data collection	Electrolytic capacitors of 10µF, 47µF
Analog-to-Digital Converter ADS1115	Jumpers
Breadboard	Light Dependent Resistor (LDR)

Set-up:

Two RC circuit are introduced to visualise and record physical quantities which characterise the charging and discharging of capacitors in RC circuits.

1. Charging and Discharging of Capacitors with Two Voltage Steps

- The first example of Raspberry Pi-based investigation with the PhyPiDAQ Software package is the charging and discharging of a 10µF electrolytic capacitor over an Ohmic resistor connected in series to the +3,3V-pin of the Raspberry Pi. The simple schematic of this RC circuit is shown in the Figure 1.
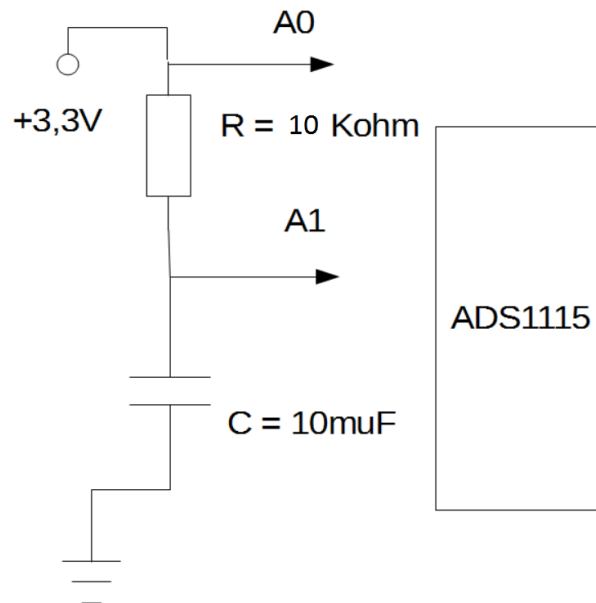

Figure 1. Charging and discharging process of an electrolytic capacitor over a resistor.

- The voltage across the capacitor U_C can be measured and converted into a digital signal with the analog-to-digital converter ADS1115 by connecting the positive lead of the capacitor to the input channel A0 and the negative lead to the ground. The potential difference U_R across the resistor can be measured as differential input between the A0 and A1 channels of the ADS1115 convertor.
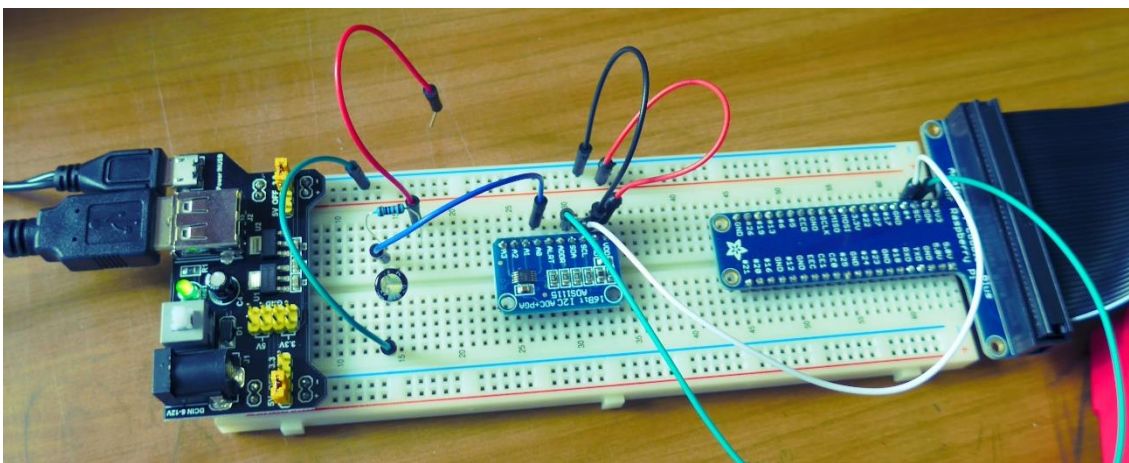

Figure 2. Charging and discharging of the capacitor by moving the free red wire between the +3,3V pin of the Raspberry Pi and the ground. The voltage across the capacitor is recorded at the A0 channel of the analog-to digital convertor.

- In order to examine the charging/discharging of the capacitor through the resistor, the students move the connecting wire (here the red wire) between the +3,3V pin of the Raspberry Pi and the ground, as shown in the Figure 2.

- The students are recommended to use the Fritzing open-source program to build this circuit diagram.

2. RC Circuit with Light Dependent Resistor (LDR) Operated Switch

- A step voltage pulse is generated at two pins of the Raspberry Pi by a light-dependent resistor (LDR) operated switch. The Fritzing-Diagram is presented in the Figure 3. Through the covering/uncovering of the LDR, the input signal at the GPIO-Pin-IN of the Raspberry Pi changes back and forth from a constant positive value to zero volts.
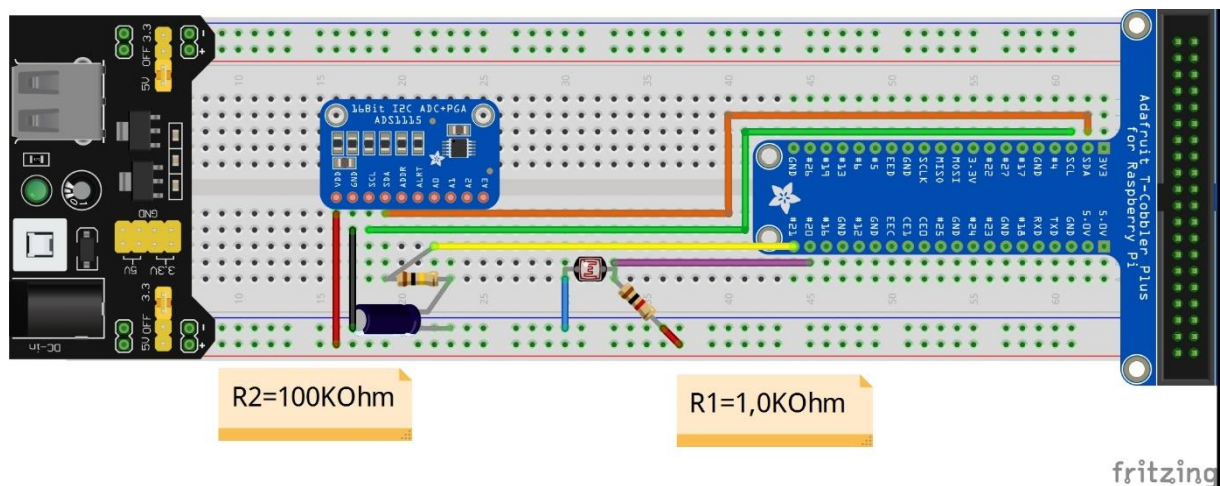

Figure 3. Fritzing-Diagram of the LDR Operated Switch to analyse the charging and discharging of an electrolytic capacitor.

- The students are recommended to draw the schematic of this RC circuit and to explain the functionality of the light-dependent resistor (LDR) operated switch in charging and discharging of the capacitor.

Safety Measures

- The electrolytic capacitor is a polarized capacitor, having the negative capacitor lead shorter and marked negative, symbolizing the flow of charge. Make sure that the negative capacitor lead is always connected to the ground, never to a positive voltage. By mixing up its polarity, the electrolyte may heat up too much or leak.
- Never connect the positive and negative pole of the Raspberry Pi directly through one wire, because that causes a short circuit which damages the Raspberry Pi permanently.

Procedure:

1. Charging and Discharging of Capacitors with Two Voltage Steps

- After constructing the circuit with the schematic in the Figure 1. the next step is to configure the experiment and the analog-to-digital Convertor ADS1115 with the PhyPiDAQ-Software.
- The first option is to use the pre-configured file `PhyPiConf_Kondensator.daq` and the ADS1115_Kondensator.yaml from the directory **/home/pi/git/PhyPiDAQ/examples**. This .daq-configuration must be reconfigured according to the `Kondensator_I_Q_2_en.daq` which is saved as .pdf file and uploaded in the **Activity-Based Physics** web page. Remember to click on `EditMode` at the top of `PhyPi Config window` to change or modify the commands in the configuration files.
- On the Graphical Interface of the PhyPiDAQ Software open the Work directory where the configuration files are going to be saved. In the `DAQ config`: text box introduce the pre-configured PhyPiConf_Kondensator.daq file from the directory **/home/pi/git/PhyPiDAQ/examples**. Rewrite this .daq-configuration according to the `Kondensator_I_Q_2_en.daq`.
- Alternatively, choose the `default.daq` file and edit this according to the `Kondensator_I_Q.daq`. which is saved as .pdf file and uploaded in the **Activity-Based Physics** web page**.** To activate the ADS1115 convertor remove the #-Sign from the command DeviceFile: ADS1115_Kondensator.yaml and press the `reload device config` button at the bottom of the `PhyPi Config` window. The `ADS1115Config_I_Q.yaml` muss be in the same directory with the .daq-configuration.
- Click on the `Device Config` Button at the top of the window to change into the Device's configuration window. Once there one can edit the .yaml configuration of the ADS1115 convertor according to `ADS1115Config_I_Q.yaml` which is saved as .pdf file uploaded in the **Activity-Based Physics** web page. The digits in the `ADCChannels: [0, 1]` together with `DifModeChan: [false, false]` set on false enable the measurement of the voltage between every of the two channels, A0 and the A1 channel, to the ground.
- After the ADS1115 convertor is properly configured, switch into the `PhyPi Config`–window to continue the experiment configuration. The `ChanLimits` are set to measure positive and negative values of the voltage on the channels A0 and A1 against the ground.
- The PhyPiDAQ-Software offers the possibility to display and record physical quantities by applying formulae to the measured voltages on the channels A0 and A1 of the ADS1115 convertor, thorough `ChanFormula` instructions. Hence the charging/discharging current is calculated by using the Ohm's law applied for the voltage across the resistor divided by the electric resistance. The resistor voltage is the difference between the measured voltages on the channels A0 and A1. The electric charge is calculated as $Q=C·U_C$, where the capacitor voltage is measured directly on the A1 channel of the ADS1115 convertor.
- To visualize the electric current and charge in real time one can chose between `DisplayModule: DataLogger` and `DisplayModule: DataGraphs.` In addition to the first module, exhibits the last one also the physical quantities at every instant of time.

2. RC Circuit with Light Dependent Resistor (LDR) Operated Switch

- After constructing the circuit shown in Figure 3. one can proceed to configure the experiment and the analog-to-digital Convertor ADS1115 with the PhyPiDAQ-Software.
- In order to generate a step voltage pulse at two pins of the Raspberry Pi, here at the 20th and the 21st , one must start the `GPIO-In_Out.py` application programmed in *Python* in the Console of the Raspberry Pi. This application one can find in the directory **/home/pi/git/PhyPiDAQ/examples** and it must run in the background.
- The values of the LDR resistance fall in bright light causing the enhancement of the voltage across the capacitor during the charging process. The response signal at the GPIO-Pins of the Raspberry Pi as well as the potential difference across the capacitor collected at the sampling rate of 0,1s, shown in Fig.4, are displayed in real-time by choosing the *DataGraphs*-Modul in the ADS1115 configuration. The collected data can be saved in .csv file for a more extensive analysis.
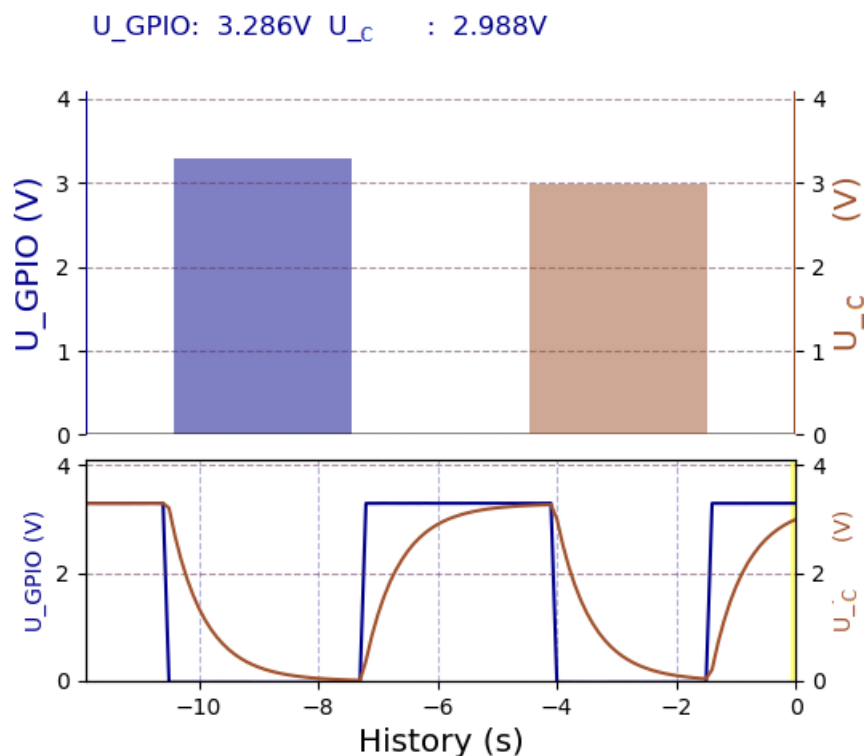

Figure 4. The step voltage pulse generated by the LDR Operated Switch at two pins of the Raspberry and the voltage across the capacitor.

- On the Graphical Interface of the PhyPiDAQ Software open the Work directory where the configuration files are going to be saved. In the `DAQ config`: text box introduce the pre-configured file `Kondensator+LDR.daq` which is saved as .pdf file and uploaded in the **Activity-Based Physics** web page. Remember to click on `EditMode` at the top of `PhyPi Config window` to change or modify the commands in the configuration files.

- To activate the ADS1115 convertor make sure that the #-Sign from the command DeviceFile: ADS1115_Kondensator.yaml has been removed and press the `reload device config` button at the bottom of the `PhyPi Config` window. The `ADS1115Config_RC_LDR.yaml` muss be in the same directory with the .daq-configuration.
- As an exercise the students can choose the `default.daq` file and edit this according to the `Kondensator+LDR.daq.`
- To visualize the electric current and charge in real time one can chose between `DisplayModule: DataLogger` and `DisplayModule: DataGraphs`. In addition to the first module, exhibits the last one also the physical quantities at every instant of time.
- After all the stages are finished, one can start the measurement by pressing the `StartRun Button` on the main graphical interface of the PhyPiDAQ. The measurement window opens. Press Run to start the measurement.
- Save the graph of the measured quantities against the time on the measurement window by pressing the `SaveGraph` Button on the taskbar. The picture called DataGraph.png is saved in the same folder containing. daq and .yaml configuration as well as the recorded measurements in the .csv file.
- Press the `End` Button to complete the measurements. To repeat the measurements, choose to don't exist the PhyPiDAQ in the coming out menu window. The same. daq configuration is meant to be used to run again the experiment, for instance for a different capacitor.

Data Collection:

- The PhyPiDAQ-Software enables measurements' collection in .csv files. In the `Kondensator+LDR.daq` or `Kondensator_I_Q_2_en.daq` configuration one has to activate Anser `DataFile: testfile.csv` as well as `CSVseparator: ';'`by removing the #-Sign. `Testfile` can be renamed to the purpose of the experiment. Add the #-Sign to disable the `DataFile: null` command. On the Raspberry Pi's Terminal, the name of the .csv file comes up.

Data Analysis:

- Perform both experiments. Observe the charging and discharging process of capacitors having different magnitudes of capacitance. In the measurement PhyPiDAQ-window watch the potential difference across the capacitor as well as the current flowing through the resistor in real-time and make predictions about their mathematical relationship with time. Use your observations as well as the recorded data to answer the following question:
- When the charging or discharging starts the potential difference across the capacitor changes rapidly, but as the time goes by, the current and the rate of rise/fall of potential difference both decrease. Explain why the rate of change of potential difference across the capacitor doesn't stay constant with time.
- In the PhyPiDAQ measurement window the duration of charging and discharging for different capacitors can be estimated. Use a torch to activate the Light Dependent Resistor (LDR) Operated Switch. The output voltage from the two pins of the Raspberry Pi changes back and forth from a constant positive value to a constant zero volts, as shown in the Figure 4.
- Vary the duration of the light pulse to visualize the length of time for charging and discharging. Explain why it takes an infinite amount of time to a capacitor to fully charge or discharge.

- Which quantities influence the charging and discharging time? To answer this question, repeat the measurement of the potential difference on the capacitor in a new RC circuit
  - a)   with a different capacitor but the same resistor;
  - b)   with a different resistor but the same capacitor;

  Which instructions in the .daq configuration of the experiment have to be revised and reedited.
- Connect two capacitors having the same capacitance in series/parallel. Observe the potential difference as function of time during the charging and discharging process. Compare the duration of charging/discharging in these cases. Explain why does it take longer in case of the capacitors connected in parallel to reach the *steady-state* condition?
- Watch the changes in the measured voltage across the two capacitors connected in series or in parallel and make prediction about the equivalent capacitance C_{eq} in each case.
- Having the potential difference on a capacitor as function of time saved on the measurement window by pressing the `SaveGraph` Button on the taskbar, as shown in the Figure 5. , describe a graphical method to determine the charging and discharging current in the RC circuit. Sketch the charging and discharging current against the time.
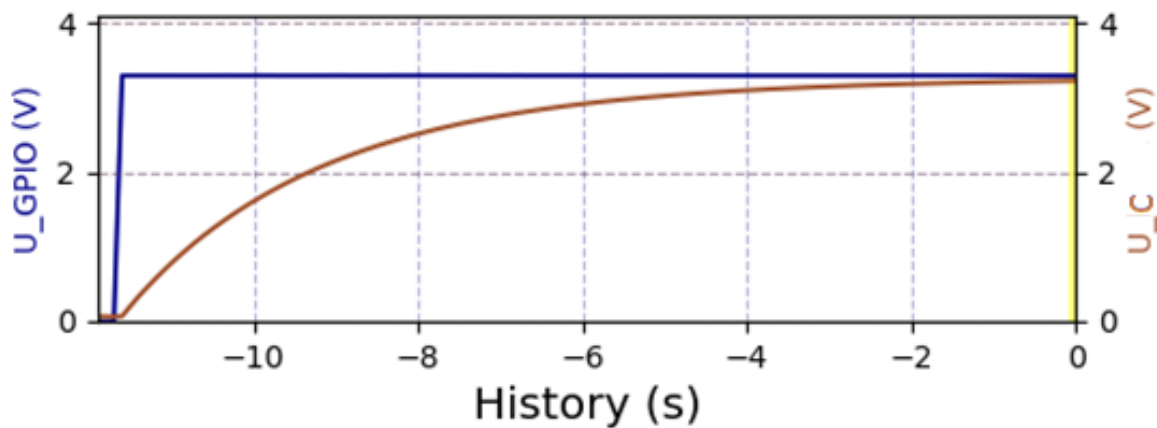
- *Figure 5. The step voltage pulse and the potential difference across an electrolytic capacitor during charging.*
- Draw a tangent to the graph of the voltage across the capacitor shown in the Figure 6. at $t_1=0s$, $t_2=2s$, $t_1=4s$, $t_1=6s$, $t_1=8s$. Compare the slops of these tangents, and interpret their falling values with time. Relate the magnitude of the slops to the charging/discharging current over time shown in the Figure 7. Why does the current decrease when the capacitor charges? Why does the current flow in opposite direction when the capacitor discharges?
- The graph of current against time shown in the Figure 6. can be used to work out the amount of charge that flows onto the capacitor by estimating the area between the graph line and the time axis. Describe a graphical method to determine the charge flowing onto the capacitor when charging. Compare your computed value to the magnitude of the electric charge one can read directly in the same Figure 7.
- The electric circuit shown in the Figure 1. allows us to measure the electric charge on the capacitor as well as the current flowing into the RC circuit. The recorded charging/discharging current flowing through the resistor in the RC circuit as well as the electric charge in the capacitor over time are shown in the Figure 6. Analyse the `Kondensator_I_Q_2_en.daq` configuration of this experiment, and answer the following questions: How are the voltage across the capacitor, electric charge and electric current mathematically related? Does this relationship change when using different capacitors?

- Interpret the directions in which the current flows during the charging/discharging process. Describe a graphical method to predict these directions when only the voltage on the capacitor vs. time is displayed.
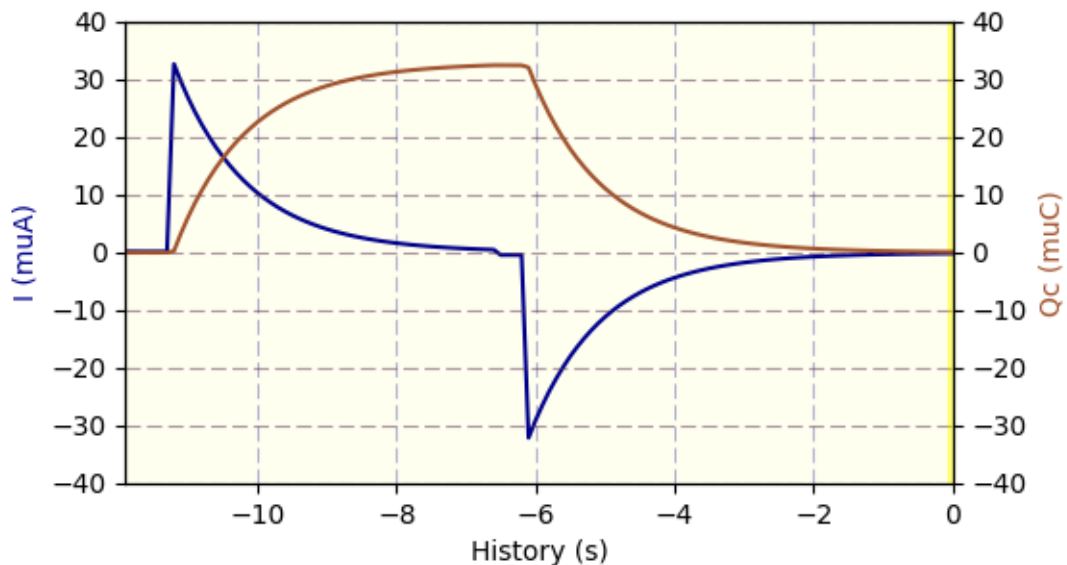

Figure 6. Charging and discharging current through the resistor as well as the charge on the capacitor of the RC-circuit in two voltage steps, recorded on the PhyPiDAQ-measurements window.

- The PhyPiDAQ-Software offers the possibility to save the measurement in .csv files, which can be used for further analysis. The `testfile_210625-0911_mod1.csv` file contains the recorded potential difference across an electrolytic capacitor of 10µF at a logging interval of 0,1s during charging and discharging when a step voltage pulse is applied. Open the file with Excel or with the LibreOffice installed on the Raspebrry Pi and compute the ratio between every two voltages recorded in equal time intervals. How do the ratio values change when compare the charging to discharging? Repeat the procedure for the measured voltages on other capacitors. How does the ratio change?
- The constant values of this ratio indicate an exponential variation of the voltage and electric charge over time. Write down the exponential functions of this quantities over time for the discharging process. Use the trendline option in Excel to draw the best fit line to the measured data. Compare your exponential function to the exponential regression of the best fit line.
- The exponential function with the base e for the voltage on the capacitor can be written as

$$U_C(t) = U_0 \cdot e^{-k \cdot t}$$

where k is the instantaneous fractional decay rate. Analyse the recoded data for different capacitors to relate k to the capacitance and resistance of the RC circuit.
- By multiplying the potential difference on the capacitor to its capacitance one can obtain the electric charge on capacitor, which has been achieved graphically thorough the `ChanFormula` in the `Kondensator_I_Q_2_en.daq` configuration. The electric charge function during the discharging process is:

$$Q(t) = Q_0 \cdot e^{-k \cdot t}$$

Derive this function with respect to time.

$$\frac{dQ}{dt} = -\frac{Q_0}{\tau} \cdot e^{-\frac{t}{\tau}}$$

Give the meaning of the minus sign. How is the electric charge on the capacitor related to the charge flowing away from the capacitor?

- Given is the equation:

$$\frac{U_R(t)}{R} = -\frac{C \cdot U_C(t)}{\tau}$$

  Explain how one can deduct this equation from the former derivative of the electric charge. Write the Kirchhoff's loop equation when the capacitor discharges, and compare it to the given equation. Determine the time constant τ.

- The time constant τ is usually obtained through the linearisation of the exponential function of the voltage across the capacitor by means of the logarithmic function. Compare the natural logarithmic function

$$\ln(U_C) = -k \cdot t + \ln(U_0)$$

  with the general form of a linear function $y = a \cdot x + b$. The log-liner plot of $\ln(U_C)$ against t results in a straight line, of slope $-k$, which crosses the $\ln(U_C)$axis at $\ln(U_0)$. Perform the logarithmic transformation of the measured voltage on the capacitor saved on the …….csv file during discharging. Plot the log-linear of $\ln(U_C)$ against t and determine the time constant. Compare it with the theoretical value calculated by using the known magnitudes of capacitance and resistance.
  Repeat the procedure to determine the capacitance or resistance by employing the logarithmic transformation to some measured data of RC circuit with different capacitors and resistors.

- An alternative procedure to determine the time constant is based on measuring the half-time $T_{1/2}$ of the discharge. This is the time it takes for the voltage on the capacitor to decrease by half. Using the measured voltage on the capacitor saved in `testfile_210625-0911_mod1.csv` file one can shift the discharging curve shown in the Figure 9. The intersection point A of the charging and discharging curves has the x-coordinate equal to the half-time, which is mathematically related to the time constant through the relationship:

$$T_{1/2} = \tau \cdot \ln(2)$$

  Determine the half-time, time constant and the capacitance if the resistance is known.
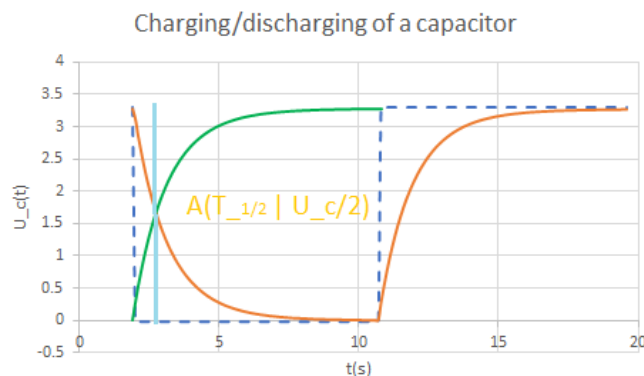

Figure 9. The measured voltage across the capacitor during charging and discharging. The shifted charging curve (green) intersects the discharging curve in A having the x-coordinate equal to the half-time.

- The RC Circuit with Light Dependent Resistor (LDR) Operated Switch facilitates also the investigation of circuits with multiple capacitors connected in series and parallel. The goal is to experimentally determine a mathematical expression that describes the equivalent capacitance of each circuit type.

  Connect two identical capacitors first in series and then in parallel. Perform the experiment with the same PhyPiDAQ- configurations. Observe and discuss the changes in the voltage across the capacitors connected in series, or in parallel in the PhyPiDAQ measurement window. Perform the logarithmic transformation of the measured voltage on the capacitor saved on the .csv file during discharging and determine the time constant for each circuit type. Compare the equivalent capacitance to the individual capacitance of capacitors, as shown in the Figure 10.
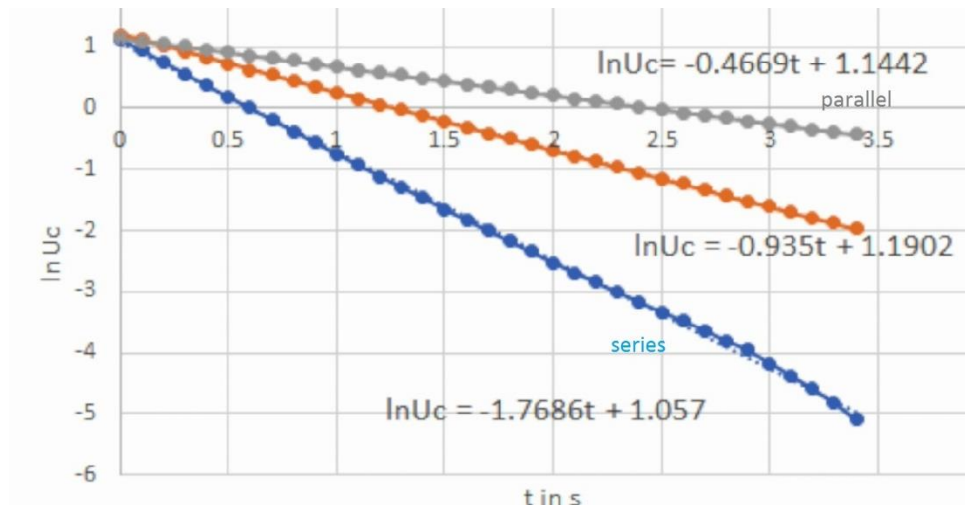

Figure 10. Log-linear of $\ln(U_C)$ against time for two identical capacitors connected in series and in parallel contrasted to the linearised voltage across one single capacitor.

- The capacitor stores energy at a magnitude which is independent of the value of the resistance. In order to prove that, one can charge the capacitor over a time longer than 5τ. The voltage across the capacitor reaches 99.3% of the maximal magnitude, so at this point the capacitor is considered to be fully charged. By discharging the energy stored in the electric field of the capacitor is converted into heat. Using the measured values of the discharging current one can compute the dissipate power on the resistor:
$$P(t) = I(t)^2 \cdot R$$

  The dissipated energy on the resistor is the time-integral of power. Open the .csv file containing the measured values of the discharging current, square these values and calculate the power at each instant of time equal to the logging time set in the .daq configuration. Compute the dissipated energy by means of numerical integration over a time period of 5τ. Compare the obtained values of the dissipated energy when the same capacitor discharges over different resistors.