Newton's Second Law

1. Objectives

- Observe and explain the relationship between the net force applied to an object and the resulting acceleration of that object.
- Design and conduct an experiment to verify the Newton's second law. The setup should consist of a Cart on a horizontal track with a string attached over a pulley to a hanging mass.
- Use the PhyPiDAQ-Software to display the position versus time graph for a Cart accelerated along the length of a track at very low friction by the weight of hanging mass.
- Observe and explain the changes in the position versus time graphs displayed in real time, if the applied force is enhanced by adding slotted weights.
- Observe and explain the changes in the position versus time graphs displayed in real time, if additional weights are loaded on the Cart while the applied force remains constant.
- Use the recorded data to determine the Cart's acceleration for each applied force.
- Use experimental data to determine the mathematical equation which relates force, mass and acceleration.
- For each experimental case, calculate the total mass multiplied by the acceleration and compare the result with the weight of hanging mass. Analyse the role of the frictional forces.

2. Materials and Equipment

Raspberry Pi 3B+ or 4B	Cart (Trolley) provided with a very low friction ball-bearing system, 500g
PhyPiDAQ-Software for Data collection	Reflector of metal, white or light colour, for reflecting the Laser beam towards the sensor's receiver
VL53L0X Motion Sensor	Jumpers to connect the sensor to the Raspberry Pi
Breadboard	Clamps
Tripod Stand	Rod
Pulley with Mounting-Rod	1.5m long Track
Weight Hanger of 10g	Slotted Weights of 10g each
Additional Mass	String

3. Set-up

The Motion Sensor is used to determine the acceleration by measuring the position versus time of the Cart of mass **m** connected with the weight hanger.

Connect the VL53L0X Motion Sensor to the Raspberry Pi according to the circuit shown in Figure 1. The students used the Fritzing open-source program to build this circuit.
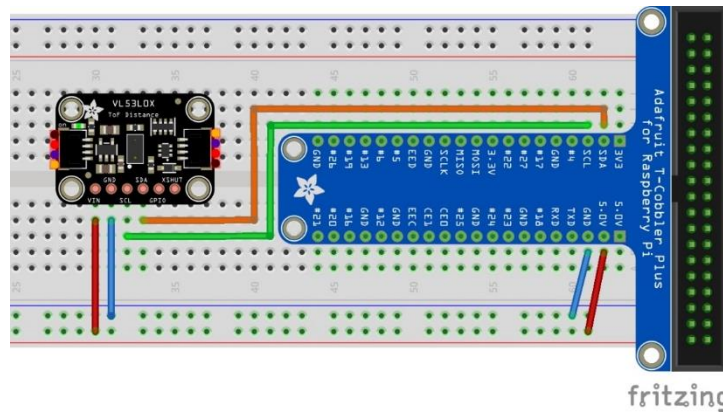
Figure 1. Connecting the VL53L0X Motion Sensor to the Raspberry Pi. The VL53L0X communicate to the Raspberry Pi via I^{2}C-Interface.

Connect monitor, mouse and keyboard to the Raspberry Pi and secure the breadboard with the sensor, as shown in the Figure 2. Or, you can use any other materials from your lab to fix the sensor into the right position to face the reflector attached to the Cart.
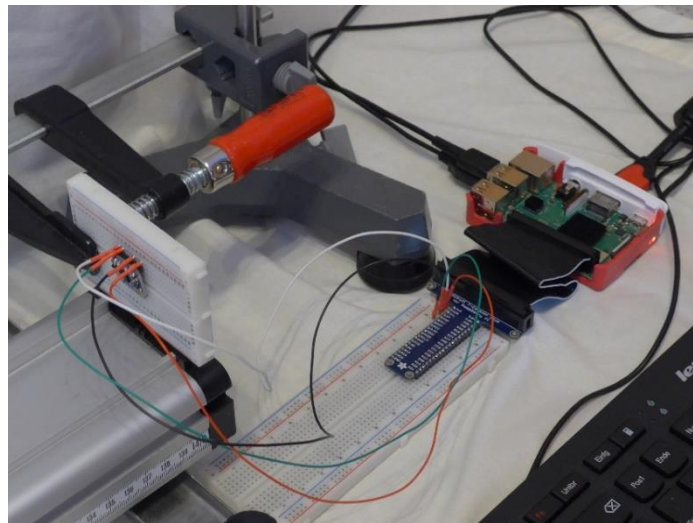

Figure 2. Connecting the VL53L0X Motion Sensor to the Raspberry Pi

Place the Cart about 3cm in front of the sensor. Position the reflector on the side facing the sensor, perpendicularly oriented to the Laser beam. According to the datasheet of the VL53L0X Motion Sensor, a light-coloured reflector provides a better quality of the measurements.

The Cart (Trolley) from the Leybold Company is provided with a very low friction ball-bearing system. It has to be placed on the track, so that it moves smoothly on the two running rails of the track. On the tray-shaped top of the Cart one can place an additional mass. The Cart is connected with the weight hanger by a string as shown in the Figure 3. The inelastic string passes over the pulley mounted with a clamp at the table's edge. The string's length has to be carefully adjusted, so that the weight hanger doesn't hit the floor before the Cart has reached the end of its run. Alternatively, one can use other Cart from other company.
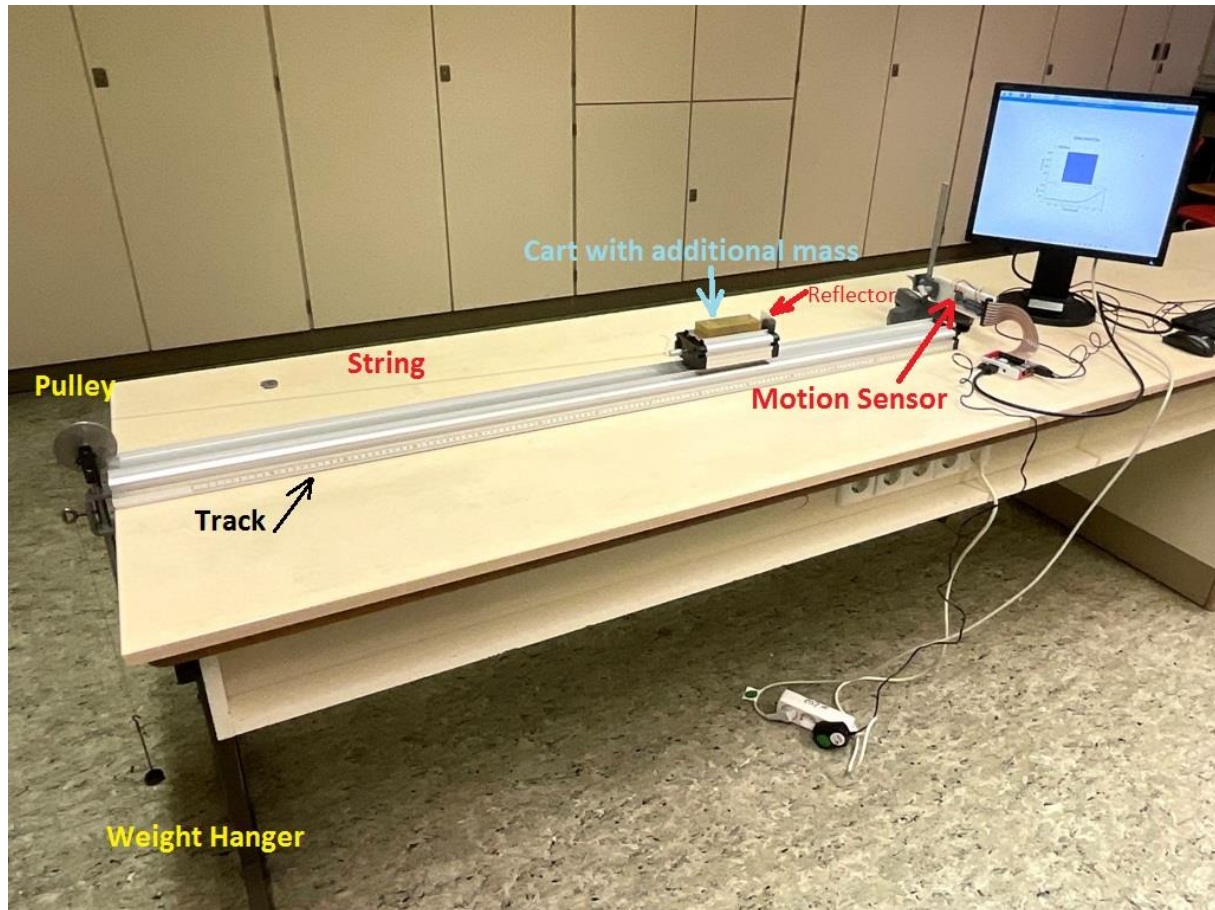
Figure 3. Set-up of the experiment with VL53L0X Motion Sensor and Cart on the track

4. Procedure

4.1 How to run the experiment

The Newton's second law will be determined by examining the motion of the Cart under the influence of a constant force, which is the weight of hanging mass. Knowing the hanging mass one can avoid to use a force sensor.

Use the balance to find the mass of the Cart and the hanging mass.

The Motion Sensor mounted on the breadboard and connected to the Raspberry Pi is used to determine the acceleration by measuring the position versus time of the Cart. The sensor has to be placed at one extremity of the track, at the appropriate height in order to face the Cart's reflector.

Before starting an experimental run, make sure that the Cart sits on the track without accelerating in either direction. If applicable, level the track by adjusting the track leveling feet.

The pulley must be clamped to the end of the track at the table's edge.

The Cart will be connected with the weight hanger. The connecting string goes over the pulley so that as the hanger with different slotted weights falls under the influence of gravity, the cart will be accelerated along the track.

Level the string by adjusting the pulley's height.

Hold the Cart steady on the track, about 3cm from of the Motion Sensor, then release it so it glides freely toward the pulley.

A student prevents the Cart from colliding with the pulley after a completed run.

The experiment will be carried out in two steps:
1. Vary the net force acting on the Cart while keeping its mass constant

- The constant force acting on the Cart is enhanced by placing additional slotted weights of 10 g on the weight hanger.
- For each hanging mass, the Cart's acceleration determined from the position vs. time graph recorded by the Motion Sensor is related to the weight of that hanging mass.
- Draw a graph showing acceleration as a function of applied force.

2. Vary the Cart's mass while keeping the force constant

- On the tray-shaped top of the Cart one can place an additional mass. If experimenting with the Cart from the Leybold Company, the additional mass has 500g.
- Place a mass of 20 grams on the weight hanger. The total hanging mass will be 30g.
- Run the experiment and determine the acceleration of the Cart with the additional mass. Compare this value to the acceleration of the unloaded Cart for the same weight of hanging mass.
- Repeat the procedure for more additional mass, while the hanging mass remains the same.
- Draw a second graph showing average acceleration as a function of the Cart's mass.

4.2 Configure the PhyPiDAQ-Software
In order to visualise the Cart's position in real time one has to go through the following steps. Watch this video https://youtu.be/Vd5KpSkuN2w first to get an insight into the procedure of configuring the experiment.

On the Graphical Interface of the PhyPiDAQ Software open the Work directory where the configuration files are going to be saved. In the `DAQ config`: text box introduce the pre-configured file `Distance.daq` from the directory **/home/pi/git/PhyPiDAQ/examples**.

Alternatively, choose the `default.daq` file and edit this according to the VL53L0x_postion_vs_time.daq saved as .pdf file and uploaded in the **Activity-Based Physics** web page. To activate the VL53L0x Motion Sensor remove the #-Sign from the command DeviceFile: config/VL53LxConfig.yaml and press the `reload device config` button at the bottom of the `PhyPi Config` window. Click on the `Device Config` Button at the top of the window to change into the Sensor's configuration window. Once there one can edit the .yaml configuration of the VL53L0x Motion Sensor according to the VL53LxConfig.yaml saved as .pdf file and uploaded in the **Activity-Based Physics** web page. Here one can choose between the two sensor types and enter the desired measuring range. Note to click on `EditMode` at the top of `PhyPi Config window` to change or modify the commands in the configuration files.

After all the stages are finished, one can start the measurement by pressing the `StartRun Button` on the main graphical interface of the PhyPiDAQ. The measurement window opens.

This video shows the experimental run with the display of the Cart's position on a horizontal track with a string attached over a pulley to a hanging mass. The experimental steps to determine the mathematical equation which relates force, mass and acceleration are emphasized https://youtu.be/F69IQdi9uS4 .

Save the Position vs. Time graph on the measurement window by pressing the `SaveGraph` Button on the taskbar. The picture called DataGraph.png is saved in the same folder containing. daq and .yaml configuration as well as the recorded measurements in the .csv file.

Press the `End` Button to complete the measurements. To repeat the measurements, choose to don't exist the PhyPiDAQ in the coming out menu window. The same. daq configuration is meant to be used to run again the experiment, for instance by setting the Motorized Cart at a higher speed.

5. Data Collection

The PhyPiDAQ-Software enables measurements' collection in .csv files. In the Distance.daq configuration one has to activate `DataFile: testfile.csv`   as well as `CSVseparator: ';'` by removing the #-Sign. `Testfile` can be renamed to the purpose of the experiment. Add the #-Sign to disable the `DataFile: null` command. On the Raspberry Pi's Terminal the name of the .csv file comes up.

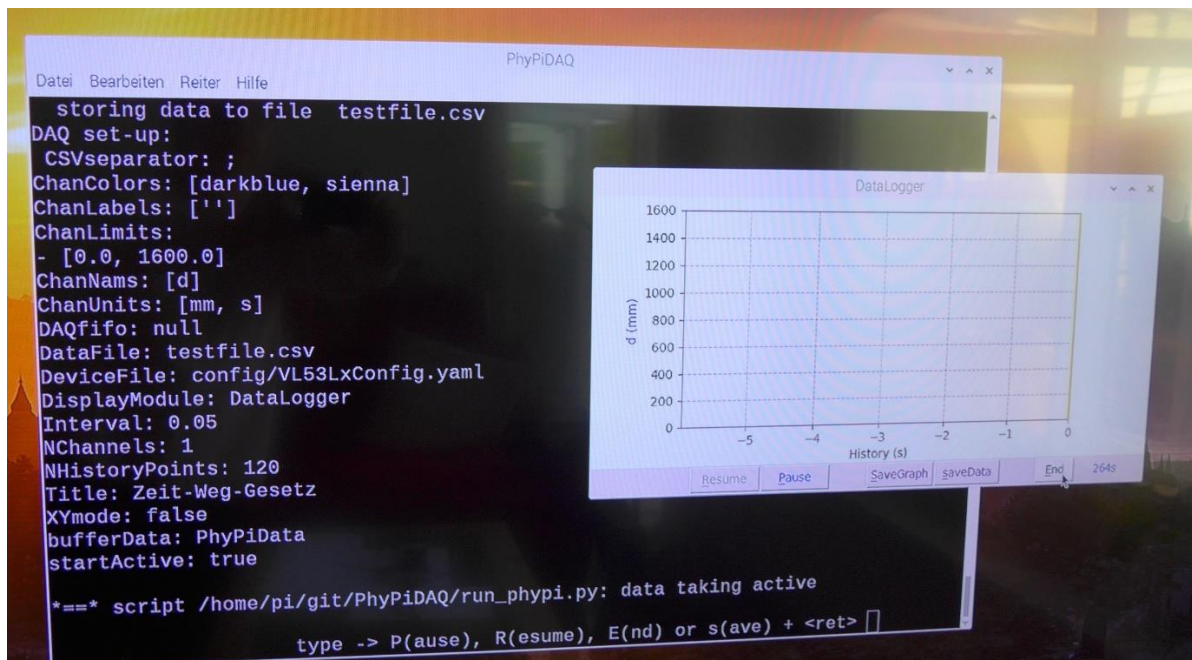Another option is to press the `SaveData` Button on the task bar of the measurement window, as shown in the Figure 4.

Figure 4. Raspberry Pi's Terminal and PhyPiDAQ measurement window. The recorded data are stored in .csv file. Alternatively, one can save data in .dat file by pressing the `SaveData` Button on the task bar of the measurement window.

6. Data Analysis

This analysis assumes a frictionless environment. Supposing that the string is inelastic, then, both the falling weight and the Cart will have the same acceleration. The acceleration is determined from the position vs. time measured with the VL53L0x Motion Sensor.

1. Vary the net force acting on the Cart while keeping its mass constant

One student will move the Cart to the end of the horizontal track until the holder with hanging mass is just under the pulley. Another student will click on the `StartRun Button` on the main graphical interface of the PhyPiDAQ, while the Cart is released. The measurement window opens. The Cart's position is displayed in real-time. The position vs. time graph can be saved by pressing the `SaveGraph` `Button` on the taskbar. The Figure 5. shows the Cart's position vs. time graph accelerated by a 20g hanging mass. Compare and interpret the change in the position vs. time graph recorded for a 40 g hanging mass, as shown in the Figure 6.
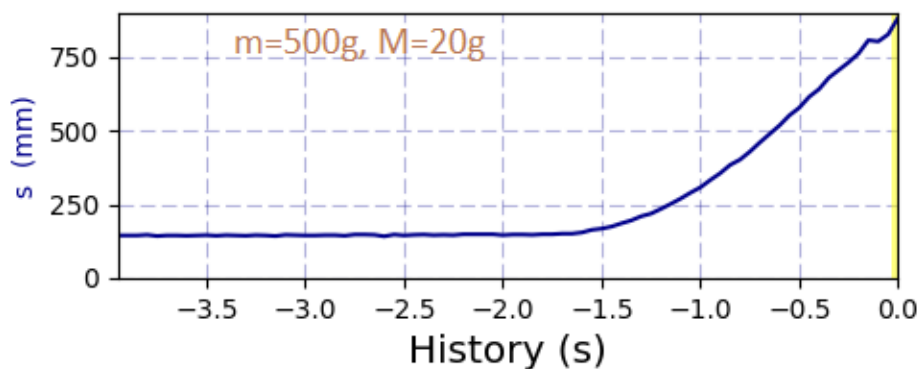

Figure 5. Position vs. time graph of the Cart with m = 500g, accelerated by a hanging mass of M = 20g

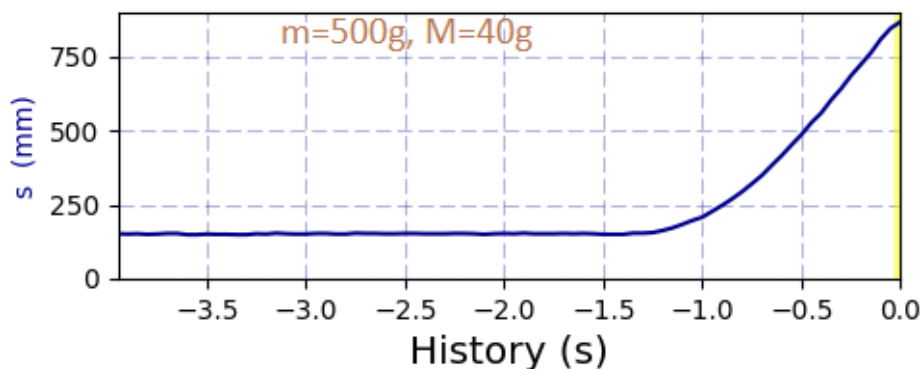*Figure 6. Position vs. time graph of the Cart with m = 500g, accelerated by a hanging mass of M = 40g*

Use the recorded data in .csv file to determine the acceleration of the Cart. Open the file with a spreadsheet program like Excel or LibreOffice, and select only the section of the graph showing the quadratic growth of the Cart's position against the time.

Select the Trendline in the scatter chart of the measured data to model the quadratic regression of the Cart's position against the time. Compare the obtained fitting function to the selected data with the distance equation for uniformly accelerated motion in one dimension, and determine the acceleration value.

The Figure 7. shows the quadratic fitting function to the data in the `m=500g_M=20g_mod.csv` file selected from the .csv file recorded for a run of the Cart having the mass m = 500g, accelerated by a hanging mass M = 20g. Determine the acceleration of the Cart by reading the coefficient of the fitting function.

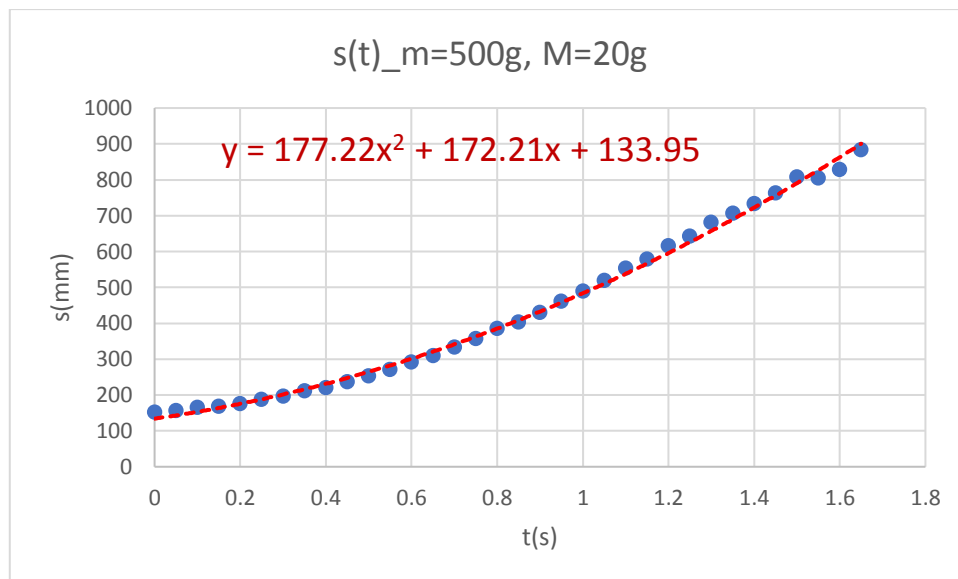**s(t)_m=500g, M=20g**

$y = 177.22x^2 + 172.21x + 133.95$

Figure 7. Position vs. time graph and the fitting function to the measurements selected from the recorded data with the PhyPiDAQ-Software, for the Cart's mass m=500g and the hanging mass M=20g

Repeat the measurement by adding one more slotted weight of 10g to the weight hanger. The hanging mass is now M = 30g. The best fit quadratic function of the selected measurements in the position vs. time file `m=500g_M=30g_mod.csv` gives the new acceleration value.

s(t)_m=500g,M=30g
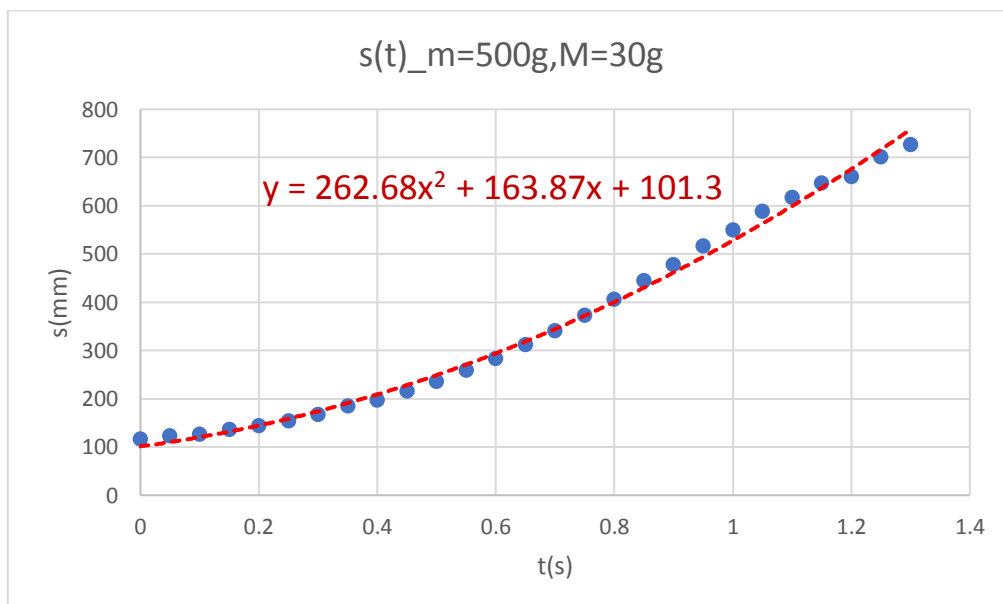
$y = 262.68x^2 + 163.87x + 101.3$

Figure 8. Position vs. time graph and the fitting function to the measurements selected from the recorded data with the PhyPiDAQ-Software, for the Cart's mass m=500g and the hanging mass M=30g

Record five trials of data using the same steps by increasing the hanging mass with 10g. For each set of unchanged values of the two masses, repeat the measurements few times, calculate the average accelerations and record in Table 1.

Table 1. Net force and average acceleration of the Cart with the mass of m= 500g

Number of trials	F(N)	a(m/s^2)

Draw a graph showing average acceleration as a function of applied force. What physical property does the slope of acceleration vs. force graph represent? What are the units of the slope?

Use your graphs to determine the relationship between applied force, and average acceleration while the Cart's mass remains unchanged.

The next important step is to verify the Newton' 2nd law. The net force acting on the system with the total mass of m_{total} = M+m is the weight of hanging mass, **F = Mg.** This force should be equal to m_{total}·**a**, if the friction is neglected.

Use the values in the Table 1. to calculate the total mass multiplied by the acceleration and record in Table 2.

Table 2. Net force and average acceleration of the Cart with the mass of m= 500g

Number of trials	F=M·g in N	a·(m+M) in N	Difference in N

As an example, consider the acceleration of the system in the Figure 7. and Figure 8. Why is the net force **F = Mg** greater than the total mass multiplied by the acceleration?

Compare the difference of the two forces in the Table 2. What means the difference **M·g - a·(m+M)**? Draw the free-body diagram for the hanging mass, M and the Cart of mass m, considering now also the frictional force. Apply Newtons 2nd law to each mass and derive **M·g - a·(m+M)**.

Why is this difference **M·g - a·(m+M)** approximately constant for different values of the hanging mass?

Calculate the ratio $\dfrac{\text{M·g} - \text{a·(m+M)}}{\text{m·g}}$ for each hanging mass and interpret the obtained values.

2. Vary the Cart's mass while keeping the net force constant

The experiment is carried out by keeping the weight of hanging mass constant.

An additional mass of 500g is placed on the Cart, which will be accelerated by a hanging mass of M=40g. Release the Cart to roll freely along the length of the track and save the position vs. time graph on the PhyPiDAQ measurement window by pressing the `SaveGraph` Button on the taskbar. Compare the position vs. time graph of the Cart with additional mass, as shown in the Figure 9. with the graph in Figure 6.
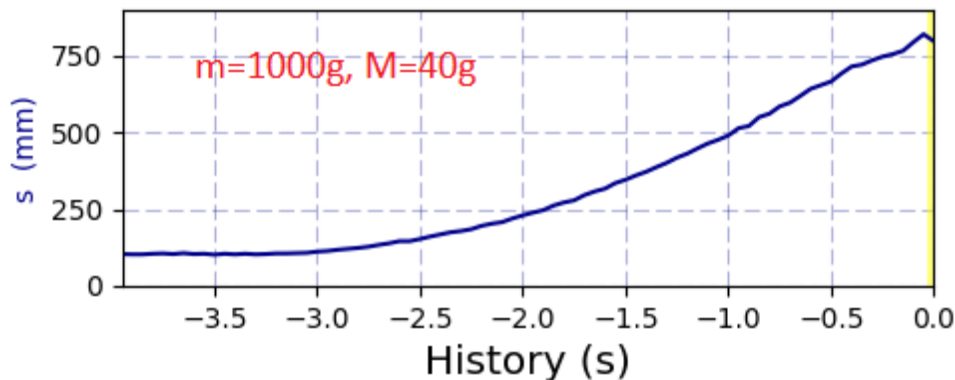

Figure 9. Position vs. time graph of the Cart with additional mass of m = 500g, accelerated by a hanging mass of M = 40g

Follow the same steps to determine the acceleration of the loaded Cart from the position vs. time graph. Open each time the .csv file with a spreadsheet program like Excel or LibreOffice, and select only the section of the graph showing the quadratic growth of the Cart's position against the time. As an example, one can open the `m=1000g_M=40g_mod.csv` file in Excel or LibreOffice and use the best fit quadratic function of the selected measurements showing a quadratic growth of the position against time, as shown in Figure 10. Determine the acceleration of the loaded Cart.
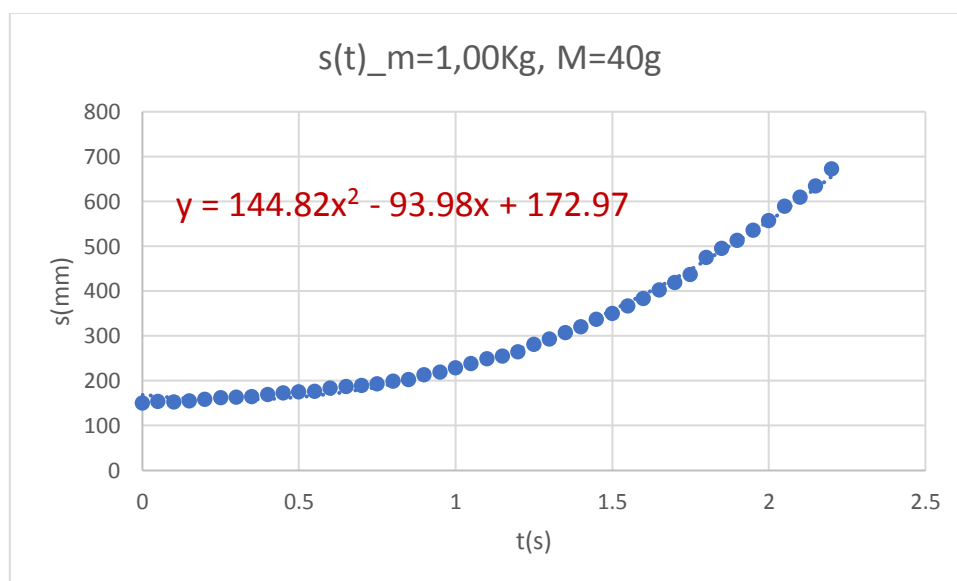

Figure 10. Position vs. time graph and the fitting function to the measurements selected from the recorded data with the PhyPiDAQ-Software, for the Cart's mass m=1000g and the hanging mass M=40g

Explain how was the Cart's acceleration different when its mass was increased while the applied net force remained constant?

Record five trials of data using the same steps by increasing the Cart's mass. For each set of unchanged values of the Cart's mass and of the hanging mass, repeat the measurements few times, calculate the average accelerations and record in Table 3.

Table 3. Cart's mass loaded with different weights and average acceleration

Number of trials	m(Kg)	a(m/s²)

Draw a graph showing average acceleration as a function of the Cart's mass. What is the shape of this graph?

Draw a second graph to analyse the relationship between acceleration and reciprocal of the Cart's mass. What physical property does the slope of acceleration vs. reciprocal mass graph represent? What are the units of the slope?

In the same coordinate system draw position vs. time graphs for the Cart with different mass values accelerated by the weight of various attached mass. Compare the graphs for different Cart's mass values, while the applied force remains constant, as shown in the Figure 11. Repeat for graphs drawn for different attached mass acting on the same Cart.
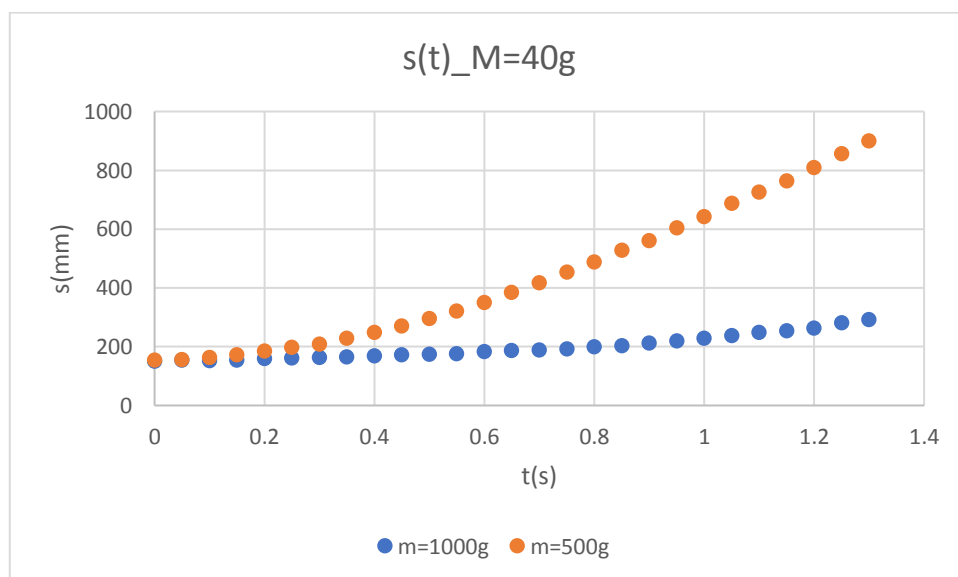Conclude the relationship between applied force, Cart's mass and acceleration.

Figure 11. Position vs. time graph for the measurements selected from the recorded data for the Cart's mass m=1000g and m=500g. The applied force is the weight of the hanging mass M=40g.

Write the mathematical equation which relates force, mass and acceleration.